# The study of estimating the parameters of a mixture of two exponential components

by

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# ABSTRACT

The problem of estimating parameters of finite mixtures, is one of the oldest estimation problems. Due to the lack of a completely satisfactory solution, this problem still attracts a great deal of attention. Other than the mixtures of normal components, the most widely used mixture distributions are the mixtures of exponential components. The simplest is the mixture of two exponential components whose probability density function is given by,

$$f(x) = p\lambda_1 e^{-\lambda_1 X} + (1-p)\lambda_2 e^{-\lambda_2 X} \quad ; \quad x > 0$$
$$= 0 \quad ; \quad \text{otherwise}$$

for $\lambda_1 , \lambda_2 > 0$ and $0 < p < 1$.

Mixtures of this type are frequently applied in life statistics and failure data. In this thesis, the problem of estimating parameters of a mixture of two exponential components is studied.

Our first effort, the use of the method of moments, did not give us satisfactory solutions. The simulation study has shown that the resulting estimates deviated drastically from the actual parameters. Next, the method of maximum likelihood

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