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University of Sri Jayewardenepura  
2011 – B.Sc. Business Administration (General) External Degree  
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BME 1503 - Business Mathematics & Statistics

**Time Allowed** : Three (3) Hours  
**Instructions** : Answer Any **FIVE** Questions  
All Questions Carry Equal Marks  
Calculators are allowed

**Question One**

(a) Simplify,  $\left(\frac{4x^{-4}y^5}{2x^{-2}}\right)^3 \times \frac{x^3y^4}{\sqrt{x^2y^2}}$ .

(b) Find the factors of the following expressions.

(i)  $8x^4y - 64xy^4$

(ii)  $ax^3 + bx - ax - b$

(c) A company manufactures three products X, Y and Z, each of which must go through three processes A, B, and C for the following times.

Product	Time spent in process		
	A	B	C
X	3	3	1
Y	3	2	3
Z	2	0	1

The maximum capacities of processes A, B and C are 130, 85 and 60 respectively. Calculate the number of units to be produced from products X, Y and Z to ensure the utilization of maximum capacity.

**Question Two**

(a) Differentiate the following functions with respect to  $x$ .

(i)  $y = \frac{6}{x^3} + 5x^{-2} - 30$

(ii)  $y = \frac{(3x^2 + 2)^3}{2x + 5}$

(b) A company invests money in a particular project and it has been estimated that after  $x$  months, the cumulated profit (in thousand rupees) from the project is given by the function  $p(x) = -3x^2 + 31.5x - 60$ , where  $x$  represents time in months. The project can run for nine months at the most.

- Draw a graph which represents the cumulative profit function.
- Calculate the break-even time points for the project.
- What is the initial cost of the project?
- Use the graph to estimate the best time to end the project.

(c) A company has found that the number of units sold of a product can be determined by the function  $S(x)$  if Rs.  $x$  (in thousand) are spent on advertising.

Where  $S(x) = -2x^3 + 27x^2 + 132x + 207$   $0 \leq x \leq 17$

- (i) How many units could be sold if no money is spent on the advertising?
- (ii) Using the differentiation find the amount to be spent on advertising to maximize the number of units sold and that maximum quantity.

**Question Three**

a) (i) Find the integral of  $\int \left( \frac{2}{x^2} - x^3 + 10 \right) dx$

(ii) Evaluate  $\int_2^4 (4 + \sqrt{x})^2 dx$

- (a) Quantity demanded and the corresponding price, under pure competition, are determined by the demand and supply functions  $p = 36 - q^2$  and  $p = 16 + \frac{q^2}{4}$ , respectively. Determine the corresponding consumer's surplus and the producer's surplus.
- (b) Amal borrowed Rs.50,000 at an interest rate of 11% compounded quarterly. At the end of each year for the next three years he agrees to make payments of Rs.15,000 each. What payment made at the end of the fourth year will extinguish the debt?

**Question Four**

- (a) Explain, how central tendency measurements and dispersion measurements important when you analyze a given set of data.
- (b) The ages of the employees of "Southern Bakers" are as follows,
  - 19, 19, 65, 20, 21, 18, 20
  - (i) Calculate the mean, median and mode of the ages.
  - (ii) Without calculating, explain how could these three measures of central location be affected if the oldest employee retires.
- (c) "AUTORUN" car rental agency recently bought 100 identical new small cars from a major car manufacturing company. After first 1000km break-in-period, obtained the following fuel consumption data.

Fuel Consumption (km per liter)	Number of Cars
9 - 10	9
11 - 12	13
13 - 14	24
15 - 16	38
17 - 18	16

- (i) Calculate the mean fuel consumption and the variance of fuel consumption for above data.
- (ii) Determine the coefficient of variation of the data.
- (iii) Which rental agency is better if coefficient of variation of fuel consumption of "FASTRUN" car rental agency is 20? Explain.

### Question Five

- (a) A manager must decide which two out of four applicants (Anil, Kamal, Sunil, Ranil) should receive job offers.
- List the sample space ( $S$ ).
  - List the sample elements in the following events.
    - Anil receives an offer
    - Kamal does not receive an offer
  - What is the probability that both Anil and Kamal receive the offer?
- (b) The number of calls received by a telephone operator between 9 a.m. and 10 a.m. has a poisson distribution with mean of 12. Find the probability that the operator receives 5 calls during the following periods.
- between 9 a.m. and 10 a.m..
  - between 9 a.m. and 9.30 a.m.
- (c) The life time of a certain brand of tyres is approximately normally distributed, with a mean of 65,000 *km* and a standard deviation of 25,000 *km* The tyres carry a warranty for 60,000 *km*.
- What proportion of the tyres that will fail before the warranty expires?
  - What proportion of the tyres will fail after the warranty but before they have lasted for 61,000 *km*?

### Question Six

- (a) "Practically sampling survey is more appropriate than complete enumeration survey (Census)" discuss on the above statement
- (b) Monthly household income in an urban area has a standard deviation of Rs.300. A random sample of 50 families reveals that the mean monthly income for this area is Rs. 20,500. Construct 95% confidence interval for mean monthly income of all families in this area.
- (c) A courier service in Colombo claims that its average delivery time is six hours for local deliveries. A random sample of the amount of time taken by this courier service for their deliveries produced the following summary measures.

$$n=16 \quad \bar{x} = 5.6\text{hours} \quad s=10 \text{ hours}$$

You are required to determine whether the given data is sufficient to support the courier's claim, at the 5% level of significance.

- State null and alternative hypotheses.
- Write down the relevant test statistic and its distribution.
- Find the critical value.
- What is your conclusion?
- What assumption must be made in order to answer above part (ii)?

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## Statistical Formulas

### Summary Measures

$$\text{Mean} = \mu = \frac{\sum fX}{N}$$

$$\sigma^2 = \frac{\sum fX^2}{N} - \mu^2$$

$$\text{Median : } h = L + \frac{\frac{N}{2} - F_{ME}}{f_m} \times C$$

$$C.V = \frac{\sigma}{\mu} \times 100$$

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times C$$

### Probability

$$\text{Conditional Probability; } P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\text{Binomial Distribution; } P(X = x) = {}^n C_x P^x q^{n-x}$$

$$\text{Poisson Distribution; } P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

### Test Statistics

For  $\mu$  when  $\sigma$  known

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

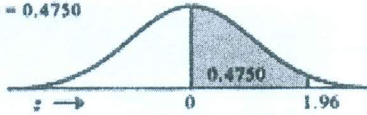
For  $\mu$  when  $\sigma$  unknown

$$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$



AREAS UNDER THE NORMAL CURVE

Example  
If  $z = 1.96$ , then  
 $P(0 \text{ to } z) = 0.4750$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4238	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

### STUDENT'S *t*-DISTRIBUTION

<i>df</i>	Level of significance for one-tailed test					
	0.100	0.050	0.025	0.010	0.005	0.0005
	Level of significance for two-tailed test					
	0.20	0.10	0.05	0.002	0.01	0.001
1	3.078	6.314	12.708	31.821	63.657	638.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.804	8.610
5	1.476	2.015	2.571	3.365	4.032	6.889
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.308	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.761
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.896	3.965
18	1.330	1.734	2.101	2.552	2.876	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291