AN APPLICATION OF THE SHORTEST PATH PROBLEM TO THE ROAD NETWORK IN SRI LANKA

by

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for the award of the PostGraduate diploma

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DECLARATION

The work described in this thesis was carried out by me under the supervision of Dr.W.B.Daundasekera and a report on this has not been submitted in whole or in part to any university or any other institution for another Degree/ Diploma.

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ABBREVIATIONS

AM	–	Ampara
AP	–	Anuradhapura
AV	–	Avissawella
BD	–	Badulla
BT	–	Batticaloa
CB	–	Colombo
GL	–	Galle
HM	–	Hambantota
JF	–	Jaffna
KT	–	Kalutara
KN	-	Kandy
KL	-	Kegalle
KG	-	Kurunegala
MN	-	Mannar
MT	-	Matale
MR	-	Matara
MG	-	Monaragala
NE	-	Nuwara Eliya
PL	-	Polonnaruwa
PT	-	Puttalam
RP	-	Ratnapura
TC	-	Trincomalee
VU	-	Vavuniya

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ABSTRACT

In this report we discuss an application to the shortest path problem. In our application we consider the road network connecting district capitals in Sri Lanka. Only class A/B roads are considered in this report. Our objective is to find the shortest route with respect to the distance from the origin to the destination while visiting other selected district capitals.

The Dijkstra's algorithm is used and coded in Visual basic 6.0 to find the shortest routes. In this report we have discussed an example to illustrate the algorithm as well as the computer program.

CHAPTER 01

Introduction

It is obvious that human resource is a key factor in developing our nation. Due to improper management, it is evident that our human resource is not utilized properly. Approximately one third of the valuable time of our nation is spent on highways. This is mainly due to wrong selection of path to the destination. The time spent on highways can be cut down by fifty percent by taking optimal paths from origins to destinations.

In this report we considered only the district capitals of Sri Lanka. The objective is to identify all possible paths from a district capital to every other district capital and implement a method to find out the optimal path in which the traveling distance is minimized.

In CAHPTER 02, a mathematical model is developed for the above problem. In network optimization theory, this problem is known as the shortest path problem. The model developed in CAHPTER 02 is a special class of a linear programming model.

CHAPTER 03 consists of the methodology of solving the linear programming problem described in CAHPTER 02. Mathematical formulation of the shortest path problem is given and has been illustrated by examples. This primal problem can be solved by solving the corresponding dual problem. The way of constructing the dual problem is also illustrated using the same example used to illustrate the primal problem. Although we can use the dual simplex algorithm to solve this problem, there exists a simple and efficient way to solve this it, called Dijkstra's algorithm which is given in CHAPTER 03. The software was developed based on the Dijkstra's algorithm.

The shortest paths obtained by simulating the computer software developed in this project is given in CAHPTER 04 . The sequence of steps leading to the optimal solution is also presented in the same chapter. The complete solution is shown in **Table 4.1**.

The advantages and limitations of this project have been discussed and possible further improvements have been suggested in CAHPTER 05 .

CHAPTER 02

Problem Definition

In network optimization theory, finding the path corresponding to optimal flow between the origin and the destination is known as the Shortest Path problem.

Suppose a person needs to travel from one district capital to another district capital in Sri Lanka in minimal possible distance. In order to find out the path corresponding to minimum traveling distance, we need to apply the shortest path problem to the road network in Sri Lanka.

Suppose we take one district capital as the origin and another one as the destination, our problem is to find out the shortest possible path between the origin and the destination.

2.1 Network

A network consists of a set of nodes linked by arcs. The notation for describing a network is (N,A), where N is the set of nodes and A is the set of arcs.

Illustration (2.1)

$N = \{1,2,3,4,5\}$

$A = \{(1,2),(1,3),(2,3),(2,5),(3,4),(3,5),(4,2),(4,5)\}$

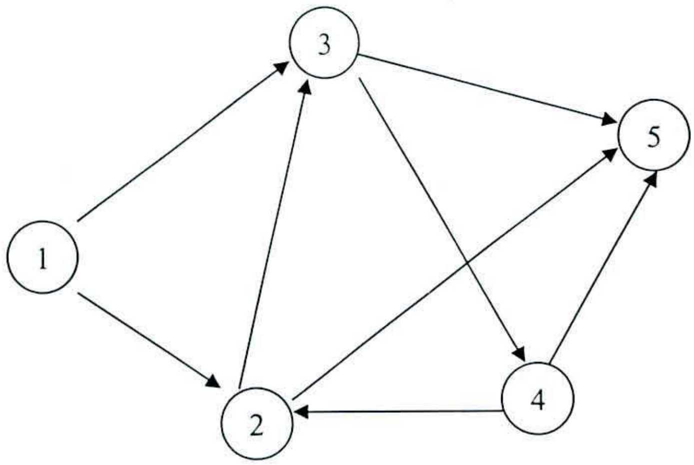

Figure 2.1 – An example to a network

An arc is said to be **directed** if it allows positive flow in one direction and zero flow in the opposite direction. If all the arcs of a network are directed, then it is called a **directed network.** A **path** is defined as a sequence of distinct arcs that joins two nodes through other nodes regardless of the direction of each arc.

2.2 Shortest path problem.

The shortest path problem determines the shortest path between origin and destination in a network.

2.3 Mathematical formulation of the shortest path problem.

The mathematical model of the shortest path problem can be formulated as follows, with respect to the following network:

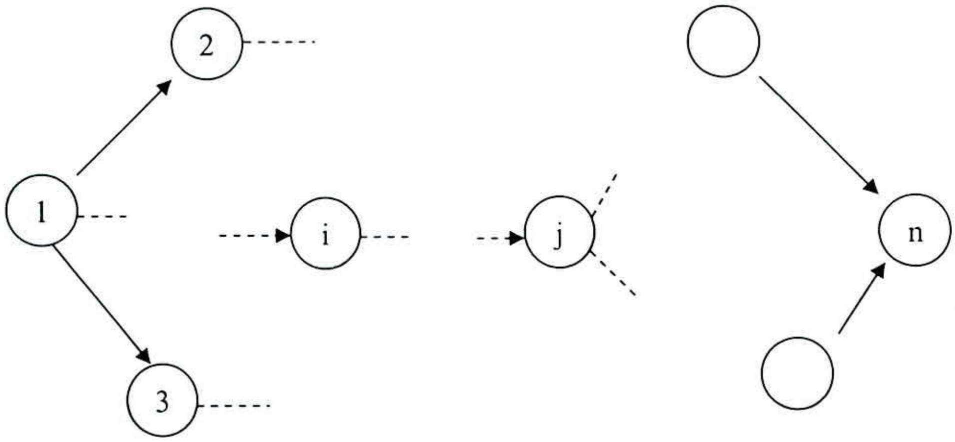

Figure 2.2 – General form of a network diagram

Suppose we need to send a single unit of flow from node 1 to node n at a minimal traveling distance.

Let x_{ij} = amount of flow in the branch *(i,j)*

c_{ij} = cost from node i to node j per unit flow.

Where $x_{ij} \geq 0, c_{ij} \geq 0 \quad i, j = 1,2,.........., n$

Accordingly, cost from node i to node $j = x_{ij}c_{ij}$

Therefore, the total cost for the network = $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$ (1)

Our objective is to minimize this function subject to the following constraints.

1. One unit of flow is emanating from node 1 and no flow is absorbed by node 1. Therefore, the potential of node 1 is 1.

i.e. $\sum_{j=1}^{n} x_{1j} - \sum_{k=1}^{n} x_{k1} = 1$ (2)